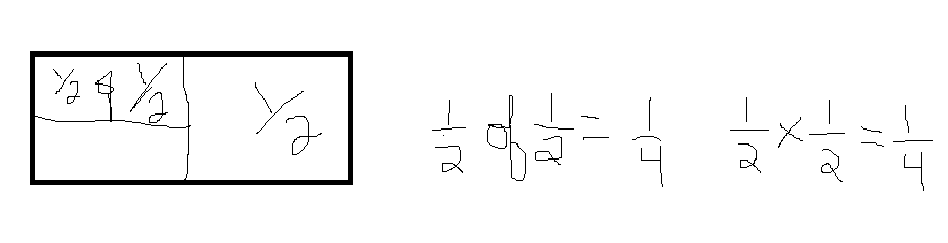
**Activities to help students visualize the multiplication of two fractions which are less than one.**

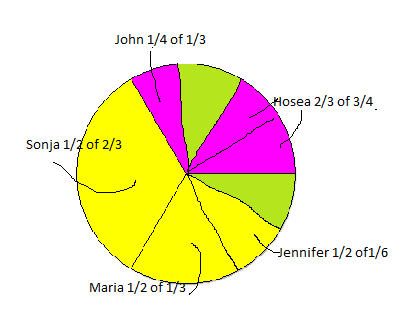
* ***Problem solving with fractions less than 1:***An excellent way for students to begin the conceptual understanding of fractions is having the students solve real world problems which they can solve without the use of an algorithm. Select several of the following problems and encourage the students to draw pictorial representations of their solutions and then as a class develop the equation for each situation.
  + You mother gives you a candy bar to split with your friend. How much would you get? Show that this be written as ½ of 1 or ½ \* 1 = ½ Another friend comes and you offer to split your ½ with your friend. How much of the whole candy bar will you now have? Show that this can be written as ½ of ½ or ½ \* ½ =1/4

Example of student picture and equations

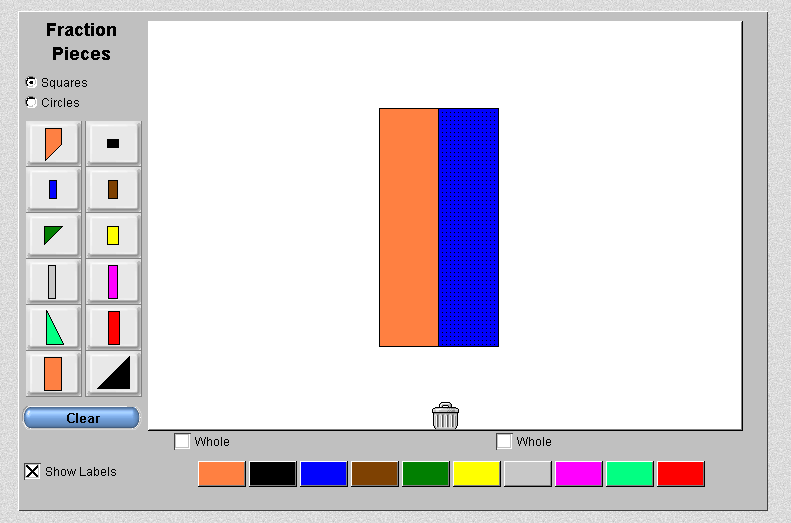


* + A cookie recipe calls for 1/3 of a cup of butter. But, you want to make half of the recipe. How much butter would you put in? ½ of 1/3 or ½ \* 1/3 = 1/6
  + You have 1/5 of a glass of water and drink ½ of the water. How much water is left in the glass. ½ of 1/5 or ½ \* 1/5 = 1/10.
  + You cut off 1/3 of the length of a rope. You then give your friend ½ of your piece. What fraction of the rope do you each have? (½ of 1/3 = 1/6)
  + You eat 1/3 of a pizza. Then your sister eats ¼ of what is left. How much of the pizza did your sister eat? ¼ of 2/3 = 1/6
  + John does 1/3 of his home work before dinner. He only get ½ of the rest done before he has to go to his soccer game. How much of his homework does he still need to finish?(1/3 + ½ of 2/3 +\_\_\_\_\_ =1 He still has 1/3 left)
  + John’s mother bakes a cake for his birthday. John’s mother gives 1/3 of the cake to John, but John only ate 1/4 of his piece. He gives what is left to Hosea who eats 2/3 of what John gave him. John’s mother gives the other 2/3 of the cake to the girls. Sonja eat ½ of the girls’ part, Maria eats 1/2 of what Sonja left while Jennifer eats ½ of what Maria left. How much of the whole cake did each person eat? How much of the cake is left? (John 1/12, Hosea 2/12 or 1/6, Sonja 1/3 or 4/12, Maria 1/6 or 2/12, Jennifer 1/12.

Student example



* ***Manipulatives:***As students began to develop a conceptualization of what it means to multiply two less than one fractions you can start to practice the process of multiplication by using manipulatives. As you use the manipulatives begin by contextualizing the problems (eg. for ½ x ¼, you might say that you have ½ of ¼ of an apple. Guide students in connecting their actions to the appropriate symbolic representation. Have the students become familiar with several manipulatives. Research has shown that students who develop multiple representations of the same concept tend to be more flexible in their thinking and retain new learnings longer.
  + Fraction Pieces or Bars (physical or virtual manipulatives) <http://nlvm.usu.edu>
    - Have the students put out the ½ piece. Ask them to find the piece that would be ½ of this piece. Write the equations ½ of ½ is ¼ or ½ \* ½ = ¼
    - Repeat for ½ of ¼, ½ of 1/6 , 1/3 of 1/4
    - Ask the students to look for the pattern (The product of the numerators and the denominators equal the numerator and denominator of the solution)
    - Have the students to put out 2/3. Ask them to find what piece would be ½ of the 2/3. ½ of 2/3 = 1/3 or ½ \*2/3 = 2/6 or 1/3

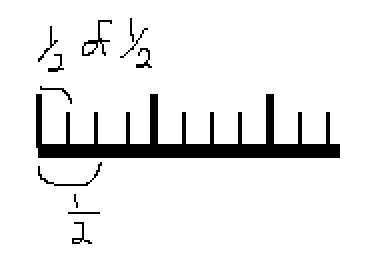


* + Number line (enlarged physical number lines or virtual manipulative – Fraction number line)

Virtual Manipulative Number Line Bars <http://nlvm.usu.edu/en/nav/frames_asid_265_g_2_t_1.html?open=activities&from=category_g_2_t_1.html>

Printable number lines <http://www.helpingwithmath.com/resources/oth_number_lines.htm>

* + - Customary Ruler: Your run ½ of a mile. Your friend ran ½ of the distance you ran. How far did your friend run? ½ x ½ = ¼ Repeat for ½ of ¼, ½ of 1/8, 1/3 of 1/6 etc.



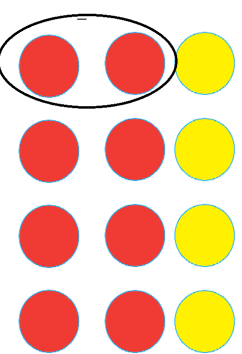
* + - Metric Ruler: You run 1/5 of a mile. Your friend ran ½ of the distance with you. How far did your friend run. ½ of 1/5 = 1/10. Repeat for ½ of 2/5, ½ of 4/5, ¼ of 4/5, 1/5 of ½
    - Ask the student if they can see a pattern. (The product of the numerators and the denominators equal the numerator and denominator of the solution)



* + Counters

Counter Printable <http://www.teachervision.fen.com/teaching-methods/printable/44640.html>

* + - Have the students put out 12 counters – turning 9 red side-up. Have them determine what fraction of the counters are red side up. (3/4). Ask them to determine how many counters would 1/3 of the ¾ be? (1/3).
    - Repeat for the following – keep a record of the sentences on the board
      * 9 counters 1/3 of 1/3
      * 6 counters ½ of 1/3
      * 10 counters 1/5 of ½
      * 8 counters ½ of ¾
      * 8 counters ¼ of ½
      * 16 counters ¼ of 1/4
      * 16 counters ½ of ¼
      * Ask the students to develop 4 sentences using 12 counters
      * Help the students see the pattern that the number of counters is a multiple of the denominators.
    - For the following problem ask students to first identify how many counters they should start with. Then have them solve the problems using counters
    - 1/3 of 3/5
    - ¼ of 1/5
    - 2/3 of ¼
    - 2/3 of ¾ ¼ of 2/3



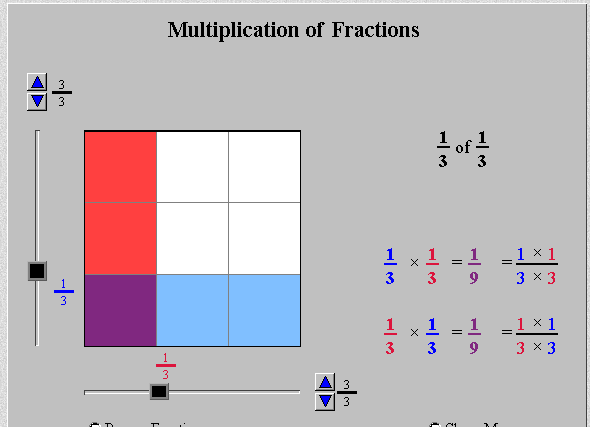
* ***Pictorial Model or Virtual Manipulatives*** – Once students have developed the conceptual understanding of the multiplication of fraction process by using manipulatives, give the students additional practice and fluency by having them do a number of pictorial model or virtual arrays.

Tool for making printable arrays <http://www2.ups.edu/community/tofu/lev2/mathconcepts/fracdecperc/mulfrac.htm>

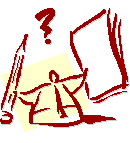
Virtual Manipulative – Fraction Rectangular Multipication

<http://nlvm.usu.edu/en/nav/frames_asid_194_g_2_t_1.html?from=category_g_2_t_1.html>

* + Using the worksheet of squares or the NLVM applet the students show the concept of multiplication by first dividing the columns to represent the first fraction and the rows to represent the second fractions. The intersection of the two models is the answer.



* + As the students are doing the pictorial models help them in see the connections to the symbolic. (e.g. 1/3 x ¼ 1x1 is the number of parts in the product, 3 x 4 is the number of parts in the whole)
* Symbolic worksheet <http://www.edhelper.com/math/fractions10.htm>
* Other activities
  + Have the students conduct a survey to find the likes of other classmates. Have them survey 24 students (or 12 if the class is small) and determine what fraction of the people are in each category (e.g. 1/3 of the people like chocolate ice cream or ¼ of the people have been to Disneyland) Then ask them to take the survey to a next level and find the fraction(e.g. 2/3 of the people who like chocolate ice cream prefer nuts on their ice cream. ¼ of the people who went to Disneyland flew there.)



* ***Assessment:*** [***http://www.math-drills.com/fractions/frac\_multiply\_reduce\_001.pdf***](http://www.math-drills.com/fractions/frac_multiply_reduce_001.pdf)

**Additional sources:**

Written information about using arrays;

<http://www.google.com/search?q=multiplication+of+fractions+arrays&ie=utf-8&oe=utf-8&aq=t&rls=org.mozilla:en-US:official&client=firefox-a>

A Utube video showing the use of arrays to multiply two fractions which are less than one.

<http://www.youtube.com/watch?v=6Zjcdw-vLNY&feature=related>

A Utube video demonstrating “Brownie Pan Model for Multiplying Fractions.

<http://www.youtube.com/watch?v=NBFS3KW_aIY&feature=related>

A Utube video demonstrating the use of counters for multiplying two fractions which are less than one.

<http://www.youtube.com/watch?v=3hxlHtMElwI&feature=channel>

A Utube video demonstrating modeling with paper folding the multiplying of two fractions which are less than one.

<http://www.youtube.com/watch?v=GFzYCTqq0MA>